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CALCULATION OF GROUNDING EFFECTS OF BARE CONDUCTORS LAID ALONGSIDE UNDERGROUND MULTI-CABLE POWER LINES BY USING ANALYTICAL EQUATIONS

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Original scientific paper

This paper presents a mathematical model to determine the fault current and voltage distribution in power cable screening conductors and bare earthing wires laid in parallel in the same cable trench. The analytical expressions for voltages and currents are derived by solving a set of differential equations. They are based on the assumption of uniformly distributed self and mutual conductances to ground of earthing wires. The earthing system under consideration includes also impedances of two adjacent substation groundings on which the earthing wires and cable screens terminate.The proposed mathematical model also takes into account mutual inductive couplings between all conductors in the trench (phases, cable screens and bare earthing wires). Particularly, in the case of a single-circuit cable line with an earthing conductor in trench, a set of generalized equations for currents and voltages along the related neutral conductors are derived in detail. An illustrative and practical example is presented too.

Keywords: *earthing, earthing wire, fault-current distribution, power cables, substation grounding potential*

Izračun uzemljivačkih učinaka golih vodiča položenih uzduž višestrukih podzemnih kabelskih vodova korištenjem analitičkih jednadžbi

Izvorni znanstveni članak

U članku je predstavljen matematički model određivanja raspodjele struje kvara i potencijala u vodičima električne zaštite energetskih kabela i u golim uzemljivačkim užetima paralelno položenim u istom kabelskom rovu. Analitički izrazi za napone i struje su dobiveni rješavajući sustav diferencijalnih jednadžbi. Oni su temeljeni na pretpostavci o jednolikoj raspodjeli vlastite i međusobnih dozemnih konduktancija uzemljivačkih užeta. Razmatrani uzemljivački sustav uključuje također i impedancije uzemljenja dviju pridruženih transformatorskih stanica na kojima završavaju krajevi uzemljivačkih užeta i kabelskih električnih zaslona. Predloženi matematički model također uzima u obzir induktivne sprege između svih vodiča u kabelskom rovu (faznih, električnih zaštita i golih uzemljivačkih užeta). Posebno su za slučaj jednostrukog kabelskog voda s jednim uzemljivačkim užetom detaljno izvedeni opći izrazi za izračun struja i napona duž neutralnih vodiča u kabelskom rovu. Predstavljen je također i jedan ilustrativan i praktičan primjer izračuna.

Keywords: *energetski kabeli, potencijal uzemljenja transformatorske stanice, raspodjela struje kvara, uzemljenje, uzemljivačko uže*

1
Introduction
Uvod

During line-to-ground faults in power systems, the zero-sequence fault current returns to the source stations through the neutral conductors (metallic cable screens, overhead ground wires, auxiliary earthing wires, etc.), or through the earthing structures and earth. Parts of the fault current, flowing into the earth through earthing structures raise the potential of the surrounding earth with respect to remote earth. Calculating an inductive influence of high-voltage cable circuits on the line-to-ground fault current distribution in related earthing systems and therefore on the substations' earthing potential rise, is of particular interest in determining the optimum design of future cable lines, as well as in evaluating the performance and reliability of existing ones.

Most modern underground power cables have an XLPE insulated core, a copper wire screen and insulated outer protecting covering (PVC or PE). Thereby, at low frequencies, cable screen shunt admitances/conductances to ground, can be neglected. These metallic screens can only be (and usually are) grounded at cable ends and therefore do not act like extra earth electrodes extending from a substation. In that case, almost the same positive earthing effects of an extended electrode can be obtained only by extending one or more auxiliary wires in the same trench and routing them closely parallel to the power lines. These bare conductors also provide better screening for nearby telecommunication circuits against inductive interference. The disturbances, which result from the line-to-ground fault current, are of concern to both the power utility and to utilities with nearby telecommunications facilities. Therefore, it is important to make an accurate analysis of the zero-sequence current distribution among the earthing system components. At the same time it is also difficult to achieve that goal, mainly due to the complexity and number of influencing factors. A number of different analytical methods have been developed in the past four decades (e.g. [1]-[4]) to accurately calculate fault current distribution. These methods yield adequate results for overhead lines but cannot be applied directly to underground cables with adjacent earthing wires.

The method described in [5], deals with current distribution along the single neutral conductor representing the underground cable sheath (or overhead shield-wire tower-footing chain). Therefore, it does not take into account mutual coupling between neutral conductors. This paper introduces an improved method of calculation of both low-frequency fault current and voltage distributions along multiple earthing (or compensation) bare conductors and an arbitrary number of insulated neutral conductors buried in the same trench. The method includes mutual inductive coupling among all the relevant conductors in a trench (phases, screens and earthing wires) as well as mutual conductive coupling among earthing wires.

2
General mathematical model
Opći matematički model

Consider two high-voltage power substations (A and B, Fig. 1) which are interconnected by power cable lines with a set of M phase conductors. These power cables have insulated metallic screens that are connected at both terminals on the appertained earthing grid.

Let the number of insulated screens be N. Furthermore,

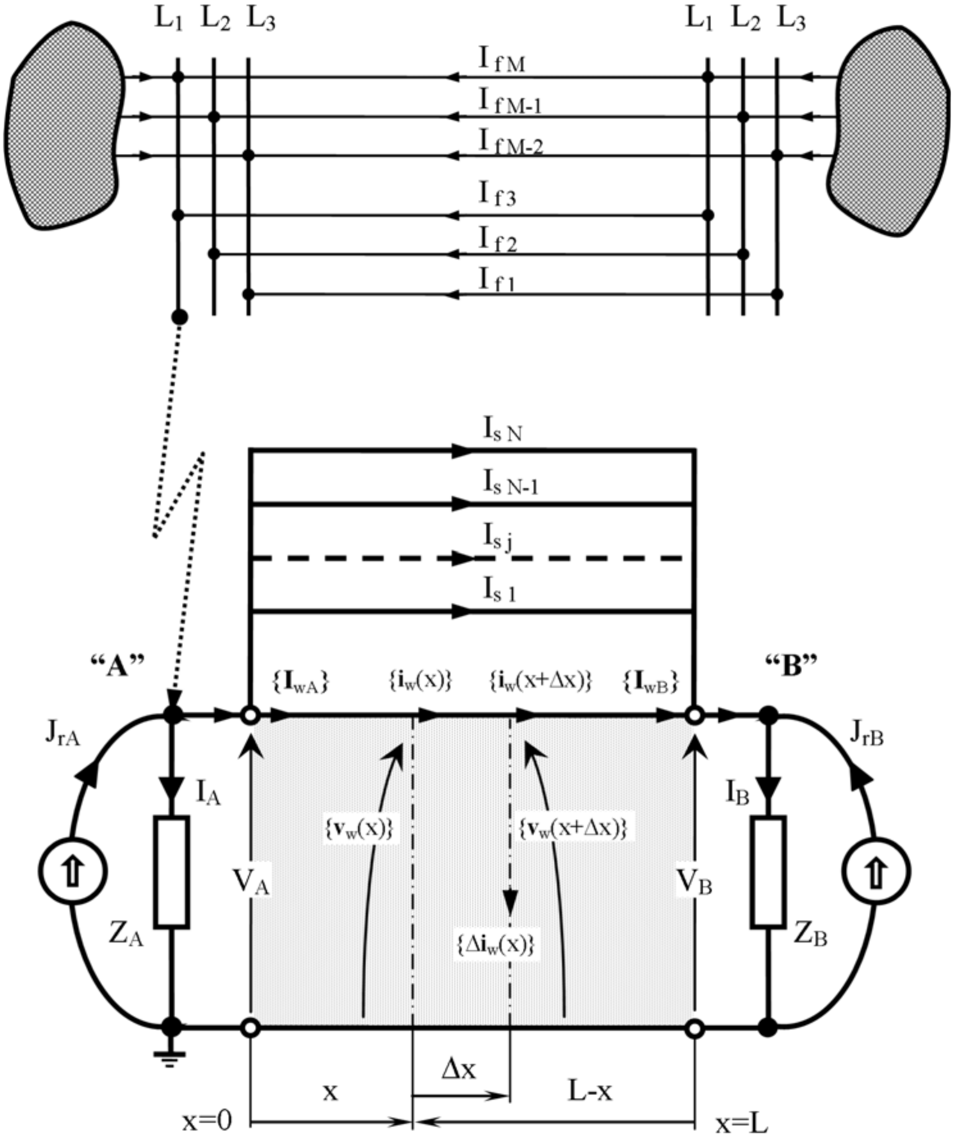

Figure 1 *Equivalent scheme of a multi-cable power line earthing system with parallel earthing wires*
Slika 1. *Nadomjesna shema sustava uzemljenja višestrukog kabelskog voda s uzemljivačkim užetima*

we presume the existence of W earthing wires laid in the same cable trench and parallel to cable screens.

The total number of connected neutral conductors is then W+N.

To investigate this case, the scheme of Figure 1 is constructed representing the zero-sequence equivalent circuit for the earthing system under consideration. The scheme also contains two zero-current sources injected into terminal substation earthings.

Referring to Fig. 1, the following set of differential equations applies to neutral conductor circuits:

$$-\frac{\mathrm{d}}{\mathrm{d}x}\{\boldsymbol{i}\}=[\boldsymbol{g}]\cdot\{\boldsymbol{v}\}, \tag{1}$$

$$-\frac{\mathrm{d}}{\mathrm{d}x}\{\boldsymbol{v}\}=[z]\cdot\{\boldsymbol{i}\}-[z_{\mathrm{f}}]\cdot\{\boldsymbol{I}_{\mathrm{f}}\}. \tag{2}$$

The axis of the power line is denoted as the x-axis, L stands for the total line length with the left end at x=0 and the right end at x=L. Denotations in (1) and (2) have the following meaning:

- $[z]$ symmetric (W+N, W+N) matrix of self and mutual unit-length serial impedances of neutral conductors, with earth return,
- $[z_{\mathrm{f}}]$ rectangular (W+N, M) matrix of mutual unit-length impedances between neutral and phase conductors, with earth return,
- $[\boldsymbol{g}]$ symmetric (W+N, W+N) matrix of self and mutual unit-length shunt (leakage) admittances of neutral conductors,
- $\{\boldsymbol{i}\}$ the vector of unknown currents $i_k(x)$ (k = 1, 2, 3,..., (W+N)) of neutral conductors at point x,
- $\{\boldsymbol{v}\}$ the vector of unknown voltages $v_k(x)$ (k = 1, 2, 3,..., (W+N)) of neutral conductors at point x,
- $\{\boldsymbol{I}_{\mathrm{f}}\}$ the vector of known phase conductor fault current values $I_{\mathrm{f}k}$ (k = 1, 2, ..., M).

By differentiating (1) and combining with (2) and analogue, differentiating (2) and combining with (1), it is obtained, respectively:

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\{\boldsymbol{i}\}-[\boldsymbol{g}]\cdot[z]\cdot\{\boldsymbol{i}\}=-[\boldsymbol{g}]\cdot[z_{\mathrm{f}}]\cdot\{\boldsymbol{I}_{\mathrm{f}}\}, \tag{3}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\{\boldsymbol{v}\}-[z]\cdot[\boldsymbol{g}]\cdot\{\boldsymbol{v}\}=\{\boldsymbol{0}\}. \tag{4}$$

The currents and voltages can be separated in two blocks according to the neutral conductor type (earthing wire or screen):

$$\{\boldsymbol{i}\}=\begin{Bmatrix}\boldsymbol{i}_{\mathrm{w}}\\ \boldsymbol{i}_{\mathrm{s}}\end{Bmatrix}, \qquad \{\boldsymbol{v}\}=\begin{Bmatrix}\boldsymbol{v}_{\mathrm{w}}\\ \boldsymbol{v}_{\mathrm{s}}\end{Bmatrix}. \tag{5}$$

so matrices $[z]$, $[\boldsymbol{g}]$ and $[z_{\mathrm{f}}]$ can be adequately partitioned into blocks, as follows:

$$[\boldsymbol{g}]=\left[\begin{array}{c|c}\boldsymbol{g}_{\mathrm{w}} & \boldsymbol{g}_{\mathrm{ws}}\\ \hline \boldsymbol{g}_{\mathrm{sw}} & \boldsymbol{g}_{\mathrm{s}}\end{array}\right], \tag{6}$$

$$[\boldsymbol{z}]=\left[\begin{array}{c|c}\boldsymbol{z}_{\mathrm{w}} & \boldsymbol{z}_{\mathrm{ws}}\\ \hline \boldsymbol{z}_{\mathrm{sw}} & \boldsymbol{z}_{\mathrm{s}}\end{array}\right], \tag{7}$$

$$[\boldsymbol{z}_{\mathrm{f}}]=\left[\begin{array}{c}\boldsymbol{z}_{\mathrm{wf}}\\ \hline \boldsymbol{z}_{\mathrm{sf}}\end{array}\right], \tag{8}$$

where

$[\boldsymbol{g}_{\mathrm{w}}]$ and $[\boldsymbol{z}_{\mathrm{w}}]$ are square (W, W) submatrices related to earthing wires only,

$[\boldsymbol{g}_{\mathrm{s}}]$ and $[\boldsymbol{z}_{\mathrm{s}}]$ are square (N, N) submatrices related to insulated metallic screens only,

$[\boldsymbol{g}_{\mathrm{ws}}]$ and $[\boldsymbol{z}_{\mathrm{ws}}]$ are (W, N) submatrices related to respective coupling between earthing wires and cable metallic screens,

$[\boldsymbol{z}_{\mathrm{wf}}]$ and $[\boldsymbol{z}_{\mathrm{sf}}]$ are (W, M) and (N, M) submatrices related to earthing wires - phase conductors and cable screens - phase conductors inductive coupling, respectively.

The calculation method presented in this paper is based on the following assumptions:

a) Uniformly distributed parameters (serial unit-length impedances $[\boldsymbol{z}_{\mathrm{w}}]$ and shunt (leakage) admittances $[\boldsymbol{g}_{\mathrm{w}}]$) are applied to the earthing wire circuit presentation.

b) Only the fundamental frequency (e.g. 50 Hz) is considered.

c) Self and mutual admittances to ground of cable screen conductors are neglected, so the submatrices $[\boldsymbol{g}_{\mathrm{s}}]$, $[\boldsymbol{g}_{\mathrm{ws}}]$ and $[\boldsymbol{g}_{\mathrm{sw}}]$ are zero-submatrices. Serial unit-length self and mutual impedances (with earth return) in $[\boldsymbol{z}]$ and $[\boldsymbol{z}_{\mathrm{f}}]$, are calculated by means of the Carson-a) Pollaczek equations [6, 7], simplified for low frequencies.

b) The medium surrounding the earthing wires and cable lines is homogeneous and characterized by average soil resistivity ρ, $\Omega\cdot$m.

c) Conductive coupling between the earthing wires and substation earthing grids is neglected.

d) The complex values of phase conductor fault currents $\{\boldsymbol{I}_{\mathrm{f}}\}$ are known from system studies.

Since the cable metallic screens are insulated from the ground, for low frequencies it may be assumed that submatrices $[\boldsymbol{g}_{\mathrm{s}}]$, $[\boldsymbol{g}_{\mathrm{ws}}]$ and $[\boldsymbol{g}_{\mathrm{sw}}]$ contain zero elements only (i.e., $g_{jk}=0$ for j,k = W+1, W+2,…,W+N). Therefore, from (1), (2) and (4) it follows, respectively:

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\boldsymbol{i}_{\mathrm{w}}\}+[\boldsymbol{g}_{\mathrm{w}}]\cdot\{\boldsymbol{v}_{\mathrm{w}}\}=\{\boldsymbol{0}\}, \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\boldsymbol{i}_{\mathrm{s}}\}=\{\boldsymbol{0}\}, \tag{10}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\boldsymbol{v}_{\mathrm{w}}\}+[\boldsymbol{z}_{\mathrm{w}}]\cdot\{\boldsymbol{i}_{\mathrm{w}}\}+[\boldsymbol{z}_{\mathrm{ws}}]\cdot\{\boldsymbol{i}_{\mathrm{s}}\}=[\boldsymbol{z}_{\mathrm{wf}}]\cdot\{\boldsymbol{I}_{\mathrm{f}}\}, \tag{11}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\boldsymbol{v}_{\mathrm{s}}\}+[\boldsymbol{z}_{\mathrm{sw}}]\cdot\{\boldsymbol{i}_{\mathrm{w}}\}+[\boldsymbol{z}_{\mathrm{s}}]\cdot\{\boldsymbol{i}_{\mathrm{s}}\}=[\boldsymbol{z}_{\mathrm{sf}}]\cdot\{\boldsymbol{I}_{\mathrm{f}}\}, \tag{12}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\{\boldsymbol{v}_{\mathrm{w}}\}-[\boldsymbol{z}_{\mathrm{w}}]\cdot[\boldsymbol{g}_{\mathrm{w}}]\cdot\{\boldsymbol{v}_{\mathrm{w}}\}=\{0\}, \tag{13}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\{\boldsymbol{v}_{\mathrm{s}}\}-[\boldsymbol{z}_{\mathrm{sw}}]\cdot[\boldsymbol{g}_{\mathrm{w}}]\cdot\{\boldsymbol{v}_{\mathrm{w}}\}=\{0\}. \tag{14}$$

The set of equations (10), directly yield the general solution vector $\{\boldsymbol{i}_{\mathrm{s}}(x)\}$ for screen currents:

$$\{\boldsymbol{i}_{\mathrm{s}}(x)\}=\{\mathrm{const.}_{\mathrm{s}}\}=\{i_{\mathrm{s1}}, i_{\mathrm{s2}}, \ldots, i_{\mathrm{sN}}\}=\{\boldsymbol{I}_{\mathrm{S}}\}. \tag{15}$$

The set of equations (13) may be solved by standard methods for linear constant coefficient differential equations to obtain the general solution vector $\{\boldsymbol{v}_{\mathrm{w}}(x)\}$ for earthing wire voltages $v_{\mathrm{w}n}(x)$ ($n=1, 2, \ldots,$ W).

In order to do that, the first step is to find the roots of the appertained characteristic equation F($r\pi$)=0 which is an algebraic polynomial equation in r of $(2\mathrm{W})^{\mathrm{th}}$ degree. For example, in most practical cases the number of earthing wires in a trench is one (W=1) or two (W=2) and the characteristic equation will be, respectively:

$$(A_{11}-r^2)=0, \tag{16}$$

$$(A_{11}-r^2)\cdot(A_{22}-r^2)-A_{12}\cdot A_{21}=0, \tag{17}$$

where A_{jk} are elements of the square matrix:

$$[\boldsymbol{A}]=[\boldsymbol{z}_{\mathrm{w}}]\cdot[\boldsymbol{g}_{\mathrm{w}}]. \tag{18}$$

Note that matrices $[\boldsymbol{z}_{\mathrm{w}}]$, $[\boldsymbol{g}_{\mathrm{w}}]$ are always symmetric, i.e.

$$[\boldsymbol{z}_{\mathrm{w}}]=[\boldsymbol{z}_{\mathrm{w}}]^{\mathrm{T}}, [\boldsymbol{g}_{\mathrm{w}}]=[\boldsymbol{g}_{\mathrm{w}}]^{\mathrm{T}}, \tag{19}$$

but the matrix $[\boldsymbol{A}]$ is symmetric only if the earthing wires are identical.

Then, the general solutions for earthing wire voltages $v_{\mathrm{w}n}(x)$, (n = 1, 2, …, W), can be written as a sum of corresponding exponential or hyperbolic functions expressed in terms of the roots r_i, (i = 1, 2,…, 2W) and (2W) arbitrary constants.

Knowing the general solution vectors $\{\boldsymbol{v}_{\mathrm{w}}(x)\}$ and $\{\boldsymbol{I}_{\mathrm{s}}\}$, one can from (11) derive the general solution vector $\{\boldsymbol{i}_{\mathrm{w}}\}$:

$$\{\boldsymbol{i}_{\mathrm{w}}\}=-[\boldsymbol{z}_{\mathrm{w}}]^{-1}\cdot\left(\frac{\mathrm{d}}{\mathrm{d}x}\{\boldsymbol{v}_{\mathrm{w}}\}+[\boldsymbol{z}_{\mathrm{ws}}]\cdot\{\boldsymbol{I}_{\mathrm{s}}\}-[\boldsymbol{z}_{\mathrm{wf}}]\cdot\{\boldsymbol{I}_{\mathrm{f}}\}\right). \tag{20}$$

Finally, the general solution vector $\{\boldsymbol{v}_{\mathrm{s}}\}$ is obtained by integrating (12) and using (18):

$$\begin{aligned}\{\boldsymbol{v}_{\mathrm{s}}\}=[\boldsymbol{z}_{\mathrm{sw}}]\cdot&\left\{\begin{array}{l}[\boldsymbol{z}_{\mathrm{w}}]^{-1}\{\boldsymbol{v}_{\mathrm{w}}\}+[\boldsymbol{z}_{\mathrm{w}}]^{-1}[\boldsymbol{z}_{\mathrm{ws}}]\cdot\{\boldsymbol{I}_{\mathrm{s}}\}\cdot x-\\ [\boldsymbol{z}_{\mathrm{w}}]^{-1}[\boldsymbol{z}_{\mathrm{wf}}]\cdot\{\boldsymbol{I}_{\mathrm{f}}\}\cdot x\end{array}\right\}-\\ &-[\boldsymbol{z}_{\mathrm{s}}]\cdot\{\boldsymbol{I}_{\mathrm{s}}\}\cdot x+[\boldsymbol{z}_{\mathrm{sf}}]\cdot\{\boldsymbol{I}_{\mathrm{f}}\}\cdot x+\{\mathbf{B}\}.\end{aligned} \tag{21}$$

Thus, each of the obtained expressions for screen voltages $v_{sj}(x)$ (j = 1, 2, …, N) can generally be written in terms of the following: the same roots r_i (i = 1, 2,…,2W), the same (2W) arbitrary constants and one additional but different arbitrary constant B_j. The total number of all arbitrary constants is then (N+2W).

A current Δi_w leaving earthing wire element of length Δx at point x is called the leakage or transversal current. A leakage current density for each earthing wire can be, by definition, calculated from (9):

$$\{\boldsymbol{i}_{dw}(x)\} = -\frac{d}{dx}\{\boldsymbol{i}_w(x)\} = [\boldsymbol{g}_w]\cdot\{\boldsymbol{v}_w(x)\}. \tag{22}$$

These currents, flowing into soil, are responsible for a local ground potential rise along the cable-line route.

Total leakage currents, cumulated along the earthing wires from terminal point x=0 to arbitrary point x, can be obtained by integrating (22):

$$\{\boldsymbol{i}_E(x)\} = \int_0^x \{\boldsymbol{i}_{dw}(x)\}\,dx = \{\boldsymbol{i}_w(\boldsymbol{0}) - \boldsymbol{i}_w(x)\}. \tag{23}$$

In special case when terminal ground potentials are neglected, solution should yield constant values of currents for all neutral conductors (even for earthing wire). These currents are exclusively a result of conductors' mutual magnetic coupling and can be derived directly from (2) substituting $\{d\boldsymbol{v}\}=\{\boldsymbol{0}\}$.

$$\{\boldsymbol{i}\} = [\boldsymbol{z}]^{-1}\cdot[\boldsymbol{z}_f]\cdot\{\boldsymbol{I}_f\}. \tag{24}$$

3 Analytical expressions for voltages and currents in neutral conductors of multi-circuit power lines with single-core cables and one earthing wire laid alongside

Analitički izrazi za napone i struje u neutralnim vodičima višestrukih vodova s jednožilnim kabelima i jednim paralelnim uzemljivačkim užetom

Applying the procedure described above to a particular case, when the number of adjacent earthing wires is W=1, the following expressions are derived:

$$[\boldsymbol{A}] = [\boldsymbol{z}_w]\cdot[\boldsymbol{g}_w] = A_{11} = z_{11}\cdot g_{11},$$

where all matrices are single-element matrices (scalars). Then, solving (16) for r, it follows:

$$r_{1,2} = \pm\sqrt{A_{11}} = \pm\sqrt{z_{11}\cdot g_{11}} = \pm\gamma.$$

The general solution expressions for earthing wire current and voltages in all neutral conductors can be written as follows:

$$i_w(x) = -\frac{1}{z_{11}}\left(\begin{matrix}\gamma\cdot C_1\cdot\cosh(\gamma x) + \gamma\cdot C_2\cdot\sinh(\gamma x) + \\ + [\boldsymbol{z}_{ws}]\cdot\{\boldsymbol{I}_s\} - [\boldsymbol{z}_{wf}]\cdot\{\boldsymbol{I}_f\}\end{matrix}\right), \tag{25}$$

$$v_w(x) = C_1\sinh(\gamma x) + C_2\cosh(\gamma x), \tag{26}$$

$$\{\boldsymbol{v}_s(x)\} = \frac{1}{z_{11}}\{\boldsymbol{z}_{sw}\}\cdot\left(\begin{matrix}\boldsymbol{v}_w(x) + \{\boldsymbol{z}_{sw}\}^T\{\boldsymbol{I}_s\}\cdot x - \\ - [\boldsymbol{z}_{wf}]\cdot\{\boldsymbol{I}_f\}\cdot x\end{matrix}\right) - \tag{27}$$
$$- x\cdot[\boldsymbol{z}_s]\cdot\{\boldsymbol{I}_s\} + x\cdot[\boldsymbol{z}_{sf}]\cdot\{\boldsymbol{I}_f\} + \{\mathrm{B}\}.$$

The arbitrary constants C_1, C_2, B_j (j= 1,2,…,N) can be evaluated if the boundary conditions are known. Thus, if the following conditions at x=0 (i.e. at the fault point "A", Fig. 1) are assumed:

$$i_w(x=0) = \mathrm{I}_{1A}, \tag{28}$$

$$v_w(x=0) = \mathrm{V}_A, \tag{29}$$

$$v_{sj}(x=0) = V_A,\ (j = 1, 2, \ldots, \mathrm{N}), \tag{30}$$

then, solving (25) to (27), the required values for these constants are obtained:

$$C_1 = \frac{z_{11}}{\gamma}\left(-I_A - [\boldsymbol{z}_{ws}]\cdot\{\boldsymbol{I}_s\} + [\boldsymbol{z}_{wf}]\cdot\{\boldsymbol{I}_f\}\right). \tag{31}$$

$$C_2 = V_A, \tag{32}$$

$$\{\mathbf{B}\} = V_A\left(\{\mathbf{1}\} - \frac{1}{z_{11}}\{\boldsymbol{z}_{sw}\}\right), \tag{33}$$

where

$\{\mathbf{1}\}$ is a unit column-vector

$[z_{ws}]=\{z_{sw}\}^T$ and $[z_{wf}]$ are row-vectors.

The final equations for voltages and currents in neutral conductors at any point "x" along the considered power line, are obtained by substituting (31) to (33) in (25) to (27):

$$v_w(x) = V_A\cdot\cosh(\gamma x) - Z_c\cdot I_{1A}\cdot\sinh(\gamma x) - \tag{34}$$
$$-\frac{\sinh(\gamma x)}{\gamma}\left([\boldsymbol{z}_{ws}]\cdot\{\boldsymbol{I}_s\} - [\boldsymbol{z}_{wf}]\cdot\{\boldsymbol{I}_f\}\right),$$

$$\{\boldsymbol{v}_s(x)\} = \frac{1}{z_{11}}\cdot\{\boldsymbol{z}_{sw}\}\cdot v_w(x) - x\cdot\left([\boldsymbol{z}_s] - \frac{\{\boldsymbol{z}_{sw}\}\{\boldsymbol{z}_{sw}\}^T}{z_{11}}\right)\cdot\{\boldsymbol{I}_s\} + \tag{35}$$
$$+ x\cdot\left([\boldsymbol{z}_{sf}] - \frac{\{\boldsymbol{z}_{sw}\}[\boldsymbol{z}_{wf}]}{z_{11}}\right)\cdot\{\boldsymbol{I}_f\} + \left(\{\mathbf{1}\} - \frac{\{\boldsymbol{z}_{sw}\}}{z_{11}}\right)\cdot V_A$$

$$i_w(x) = -\frac{V_A}{Z_c}\cdot\sinh(\gamma x) + I_{1A}\cdot\cosh(\gamma x) + \tag{36}$$
$$+\frac{\cosh(\gamma x) - 1}{z_{11}}\cdot\left([\boldsymbol{z}_{ws}]\cdot\{\boldsymbol{I}_s\} - [\boldsymbol{z}_{wf}]\cdot\{\boldsymbol{I}_f\}\right),$$

where:

$$Z_c = \sqrt{\frac{z_{11}}{g_{11}}}.$$

Next, substituting the boundary condition values at $x=L$ (point "B", Fig. 1), i.e.:

$$i_w(x=L) = I_{1B}, \tag{37}$$

$$v_w(x=L) = V_B, \tag{38}$$

$$v_{sj}(x{=}L) = V_B,\ (j = 1, 2, \ldots, \mathrm{N}), \tag{39}$$

in (34) to (36), the following set of (N+2) linear equations, combining the values of neutral conductor voltages and currents at both terminals, is obtained:

$$V_B = V_A \cdot \cosh(\gamma L) - Z_c \cdot I_{1A} \cdot \sinh(\gamma L) - \\ - \frac{\sinh(\gamma L)}{\gamma}\left(\left[z_{ws}\right]\cdot\left\{I_s\right\} - \left[z_{wf}\right]\cdot\left\{I_f\right\}\right), \tag{40}$$

$$\left\{V_B\right\} = \frac{1}{z_{11}}\cdot\left\{z_{sw}\right\}\cdot V_B - L\cdot\left(\left[z_s\right] - \frac{\left\{z_{sw}\right\}\left\{z_{sw}\right\}^T}{z_{11}}\right)\cdot\left\{I_s\right\} + \\ + L\cdot\left(\left[z_{sf}\right] - \frac{\left\{z_{sw}\right\}\left\{z_{wf}\right\}}{z_{11}}\right)\cdot\left\{I_f\right\} + \left(\left\{1\right\} - \frac{\left\{z_{sw}\right\}}{z_{11}}\right)\cdot V_A, \tag{41}$$

$$I_{1B} = -\frac{V_A}{Z_c}\cdot\sinh(\gamma L) + I_{1A}\cdot\cosh(\gamma L) + \\ + \frac{\cosh(\gamma L) - 1}{z_{11}}\cdot\left(\left[z_{ws}\right]\cdot\left\{I_s\right\} - \left[z_{wf}\right]\cdot\left\{I_f\right\}\right). \tag{42}$$

Usually, the known values are only phase currents coming, during the phase-to-ground fault, from all substation transformers and power lines connected to buses L_1, L_2 and L_3 of substations "A" and "B".

Since the number of equations in (40) to (42) is (N+2) and the number of the unknown quantities (V_A, I_{1A}, V_B, I_{1B} and $\{I_s\}$) is (N+4), it is necessary to define two additional boundary relations. The simplest way to do so, applied frequently in practice, is to presume the zero values for both substation earthing voltages (V_A=0 and V_B=0). These zero values are often unrealistic substitutes for exact values. Therefore, in this paper more realistic additional boundary conditions for terminal voltages are introduced:

$$V_A - Z_A I_A = 0, \tag{43}$$

$$V_B - Z_B I_B = 0, \tag{44}$$

where Z_A and Z_B are the terminal earthing impedances at "A" and "B" (Fig. 1), respectively, with neutral interconnection conductors excluded.

Next, in order to achieve more realistic results (as recommended in the harmonization document HD 637 S1 approved by CENELEC and in [8]), the "reduced fault currents" J_{rA} and J_{rB} should be calculated and injected at earthing nodes "A" and "B", instead of the total phase-to-ground fault current injected at the fault point. These currents which actually enter the ground through earthing structures connected at points "A" and "B" respectively, are calculated separately using the cross-section data and zero-sequence (or phase) currents of power lines incident to substations "A" and "B".

Therefore, the boundary conditions should be completed with current balances at x=0 and x=L. According to Fig. 1, these conditions can be written as:

$$I_A + I_{1A} + \sum_{j=1}^{N} I_j = J_{rA}, \tag{45}$$

$$I_B - I_{1B} - \sum_{j=1}^{N} I_j = J_{rB}. \tag{46}$$

Finally, to obtain the solution vector $\{X\}$ for unknown values of voltages and currents at both terminals, one has to solve the set of (N+6) simultaneous linear equations (40) to (46) which can be compactly written in the following matrix form:

$$\left[K\right]\cdot\left\{X\right\} = \left\{\mathbf{b}\right\}, \tag{47}$$

and, hence,

$$\left\{X\right\} = \left[K\right]^{-1}\cdot\left\{\mathbf{b}\right\}, \tag{48}$$

where

$$\left\{\mathbf{X}\right\} = \left\{V_A \quad I_{1A} \;\vdots\; V_B \quad I_{1B} \;\vdots\; I_A \quad I_B \;\middle|\; \left\{I_s\right\}^T\right\}^T, \tag{49}$$

$\{I_s\}$ – the unknown column vector of N screen currents,
V_A, V_B, I_{1A}, I_{1B} – the unknown earthing wire terminal voltages and currents,
I_A, I_B – unknown substation (A and B) earthing structure currents,
$[K]$ – the coefficient matrix of equation system (40) to (46),
$\{\mathbf{b}\}$ – the column vector of equation system constant terms.

Equations for a leakage current density and cumulative earth current, derived from (22) and (23), are respectively:

$$i_{dw}(x) = g_{11}\cdot v_w(x), \tag{50}$$

$$i_E(x) = I_{1A} - i_w(x), \tag{51}$$

where, for $v_w(x)$ and $i_w(x)$, expressions (34) and (36) are to be inserted respectively.

4 Illustrative example
Ilustrativan primjer

As an example of the mathematical model application we have developed, consider a cable line interconnecting substations A and B with a given fault location at x=0 (see Figures 1 or 2) and the appertained cable trench of Fig. 3, in which the following conductors exist:

- One group of three single-core 110 kV cables forming a three-phase power line,
- One bare earthing wire (Cu, 50 mm^2) extended at depth H= 0,7 m,
- Cable type: single-core 110 kV, (ABB AXKJ 1000 mm^2, Al, with a cable screen 95 mm^2, Cu); cable length L = 3000 m,
- D = 0,084 m; D_1 = 0,5 m; D_2 = 0,35 m; H = 0,7 m.

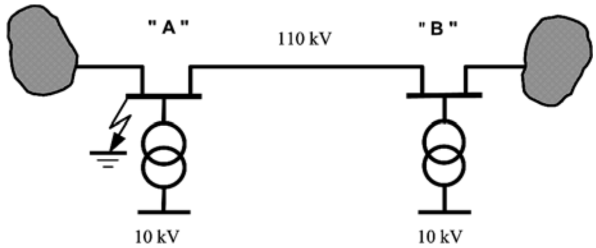

Figure 2 *Single-phase diagram of a cable line interconnecting substations "A" and "B"*
Slika 2. *Jednopolna shema kabelskog spoja između TS "A" i TS "B"*

Other relevant input data are:

- Phase fault currents: $I_{f1} = 13160\angle{-74{,}97}\ °A$; $I_{f2} = 226\angle 85{,}95\ °A$; $I_{f3} = 226\angle 85{,}98\ °A$
- Injected terminal currents: $J_{rA} = 14810{,}6\angle{-92{,}1}\ °A$, $J_{rB} = 2161{,}6\ \angle 91{,}98\ °A$.
- Soil resistivity: 200 Ω·m.
- Earthing conductor's self leakage admittance: g_{11}=0,00149 S/m
- Substation earthing impedances: $Z_A = 0{,}2\ \Omega$, $Z_B = 1{,}0\ \Omega$.

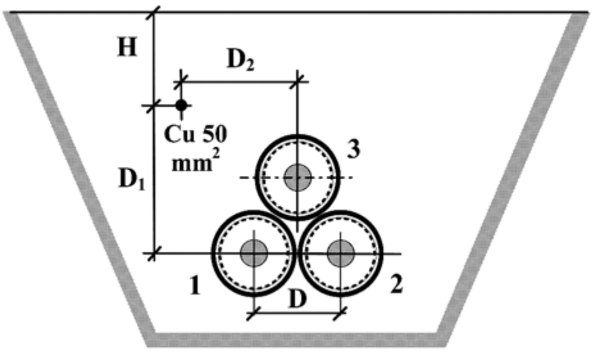

Figure 3 *An example of cross-section through the cable trench with three 110 kV single-core cables and one earthing conductor*
Slika 3. *Primjer poprečnog presjeka kabelskog rova s 3 jednožilna kabela 110 kV i jednim uzemljivačkim užetom*

To illustrate effects of laying a bare wire alongside the underground cable line, the following three cases are calculated:

a) earthing grids of substations A and B are interconnected by cable-line screens only (i.e. W=0, $[z] = [z_s]$, $[z_f] = [z_{sf}]$).
b) earthing grids of substations A and B are interconnected both by cable-line screens and one earthing wire.
c) the same as case b) but the effect of inductive coupling between the cable screens and the wire, is neglected (i.e. $[z_{sw}] = [z_{ws}]^T = [\mathbf{0}]$).

The main calculated results are presented in Tab. 1. Comparing the numerical results of the above three cases, it becomes obvious that laying an earthing wire and connecting it to substation earthing grids, significantly helps in reduction of earthing grid potentials of both substations.

Table 1 *Calculation results*
Tablica 1. *Rezultati izračuna*

	a) W=0 $\{z\} = \{z_s\}$	**b)** W=1	**c)** W=1 $\{z_{sw}\} = \{\mathbf{0}\}$
V_A/V	549∠-125°	408∠-122°	452∠-119°
V_B/V	219,7∠-150°	110,4∠-136°	108,8∠148°
I_{1A}/V	0	1936∠-118°	597,2∠-151°
I_{1B}/V	0	1408∠-103°	80,4∠-56,3°
I_{S1}/V	4842∠-50,1°	4642∠-45,6°	4849∠-49,9°
I_{S2}/V	3867∠-77,8°	3536∠-73,9°	3866∠-77,5°
I_{S3}/V	3867∠-77,8°	3500∠-74,4°	3866∠-77,5°
I_A/A	2745∠-125°	2042∠-122°	2262∠-119°
I_B/A	219,7∠-150°	110,4∠-136°	108,8∠148°

Furthermore, at the same time this wire acts as a compensating conductor which reduces the magnitudes of cable screen currents.

Therefore, the earthing wire connection is always in favor of both the substation earthing voltage safety and cable thermal withstanding.

Fig. 4 graphically illustrates magnitude distributions of both, the cable screen ($v_{s1}(x)$) and earthing wire ($v_w(x)$) potentials for the case b).

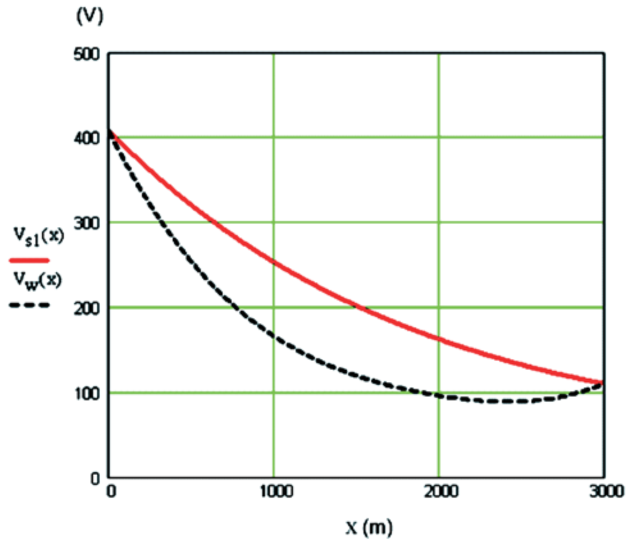

Figure 4 *Calculated potential profiles of cable screen and earthing wire (for the case b)*
Slika 4. *Izračunati profili potencijala kabelskog električnog zaslona i uzemljivačkog užeta (za slučaj b)*

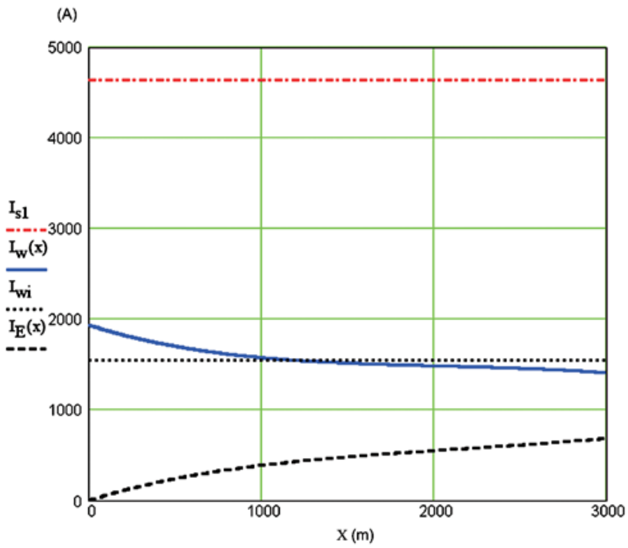

Figure 5 *Earthing wire current profiles longitudinal (I_w) and cumulative leakage (I_E)*
Slika 5. *Promjena iznosa uzdužne (I_w) i kumulativne poprečne (I_E) struje*

For the same case and given fault location, Fig. 5 graphically illustrates magnitude distributions of both the earthing wire longitudinal current ($I_W(x) = |i_w(x)|$) and cumulative leakage (earth) current ($I_E(x) = |i_E(x)|$). The maximum screen current (I_{S1} = 4642A.), shown also in Fig. 5, flows in the screen of the faulty-phase cable (denoted as No. 1 in Fig. 3).

In a particular case when $Z_A = Z_B = 0$, a solution of (48) always yields the constant values of currents for all neutral conductors (i.e. including the earthing wires). These currents must have identical values as currents derived using (24). In this special case ground potential rise does not exist, $i_E(x)$=0 and the earthing wire has only "induced" constant current of magnitude $I_{wi} = |i_{wi}| = 1546$ A (see Fig. 5).

Fig. 6 graphically illustrates (for the case b), the magnitude distribution of the earthing wire leakage current density $|i_{wd}(x)|$ and its average value I_{wdsr} which is obtained from:

$$I_{wdsr} = \frac{|i_E(L)|}{L} = \frac{|I_{1A} - I_{1B}|}{L} = 0{,}228 \text{ A/m}.$$

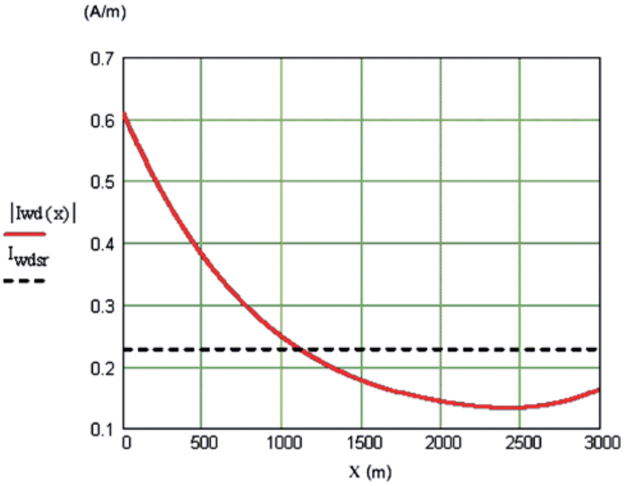

Figure 6 *The earthing wire leakage current density profile $|i_{wd}(x)|$ and the average leakage current density I_{wdsr}*
Slika 6. *Promjena iznosa gustoće poprečne struje $|i_{wd}(x)|$ duž uzemljivačkog užeta i prosječna gustoća poprečne struje I_{wdsr}*

5 Conclusion
Zaključak

This paper has presented a set of equations providing a more accurate calculation of the fault current and related voltage distributions in neutral conductors of underground cable lines interconnecting two neighboring substations and the accompanied parallel earthing wires. The paper outlines the effects of inductive coupling between cable neutral conductors and earthing wires on both current and voltage distributions in the earthing system during phase-to-ground fault. The accurate determination of earthing wire and cable screen currents under fault conditions is important to insure proper transmission line design and to select proper number and size of earthing wires.

A detailed parametric analysis, appplied to the illustrative example, has shown that an earthing wire terminal connection to the substation earthing is always in favor of both the terminal substation safety and the cable thermal withstanding. This connection also provides better screening for nearby auxiliary cables against inductive interference. The mathematical model derived is valid for stationary fault conditions and low frequencies.

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